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50. Proposed by SYLVESTER ROBINS, North Branch Depot, New Jersey.

The edges of a rectangular parallelopiped are within 1 of the proportion 2 : 3 : 9, and they are $2x \pm 1$, $3x$ and $9x$, $(2x \mp 1)^2 + (3x)^2 + (9x)^2 =$ the diagonal squared $= 94x^2 \mp 4x + 1 = \square$. To find four integral values for x .

I. Solution by A. H. HOLMES, Box 963, Brunswick, Maine.

We may put it in the form : $90x^2 + (2x \pm 1)^2 = \square$, or
 $m^2x^2 - (m^2 - 90)x^2 + (2x \pm 1)^2 = \square$.

$\therefore 2m(2x \pm 1) = (m^2 - 90)x$; $4mx \pm 2m = m^2x - 90x$.

$\therefore x = \pm(2m/(m^2 - 4m - 90))$.

Let $m = nx$. Then $n^2x^2 - 4nx - 90 = \pm 2n$; $n^2x^2 - 4nx + 4 = 94 \pm 2n$.

Take plus sign and let $n = 3$. $\therefore 3x = 2 + 10 = 12$. $\therefore x = 4$.

Now let $n = a/b^2$. $a^2x^2/b^4 - 4ax/b^2 + 4 = 94 \pm 2a/b^2 = (94b^2 \pm 2a)/b^2$.

Now take $b = 3$. $\therefore a = 5/2$ and $a/b^2 = 5/18$.

$\therefore 5x/18 = 2 + 29/3$. $5x = 36 + 174 = 210$. $\therefore x = 42$.

Now let $b = 10$. $\therefore a = 9/2$ and $a/b^2 = 9/200$.

$\therefore 9x/200 = 2 + 97/10$. $9x = 400 + 1940 = 2340$. $\therefore x = 260$.

Now let $b = 23$. $\therefore a = -3/2$ and $a/b^2 = -3/1058$.

$\therefore -3x/1058 = 2 - 223/23$, or $3x = 8142$. $\therefore x = 2714$.

For $x = 4$ we have : $94x^2 + 4x + 1 = \square$.

For $x = 42$ we have : $94x^2 - 4x + 1 = \square$.

For $x = 260$ we have : $94x^2 + 4x + 1 = \square$.

For $x = 2714$ we have : $94x^2 - 4x + 1 = \square$.

II. Solution by A. H. BELL, Hillsboro, Illinois.

The equation readily reduces to : $t^2 - 94y^2 = -90$(1),

and $x = (t \mp 2)/94$ (2). (1) $\div 9$ gives $t'^2 - 94y'^2 = -10$, and
 $t = 3t'$, $y = 3y'$(3).

One cycle.

$\sqrt{94}$. No. of Frac's:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16:
Quotients complete Denom'rs	1	13	6	5	9	10	3	15	2	15	3	10	9	5	6	13 1:
Quotients	9	1	2	3	1	1	5	1	(8)	1	5	1	1	3	2	1 18:
Convergents $\frac{t}{y}$	$\frac{1}{0}$	$\frac{2}{13}$	$\frac{3}{6}$	$\frac{5}{5}$	$\frac{9}{9}$	$\frac{10}{10}$	$\frac{3}{3}$	$\frac{15}{15}$	$\frac{2}{2}$	$\frac{15}{15}$	$\frac{3}{3}$	$\frac{10}{10}$	$\frac{9}{9}$	$\frac{5}{5}$	$\frac{6}{6}$	$\frac{13}{13}$

The convergents preceding the denominators, 10 of the complete quotients $= t'/y' = 126/13$ and $85038/8771$ (4), as they are even fractions.

\therefore answer the -10 of (3). To obtain other values of t' and y' , take $v^2 - 94u^2 = 1$(5).

$$(3) \times (5) \text{ and } \pm 188t'uvy' \quad (t'v \pm 94uy')^2 - 94(t'u \pm vy')^2 = -10 \quad \left. \vphantom{\begin{matrix} (3) \times (5) \text{ and } \pm 188t'uvy' \\ (t'v \pm 94uy')^2 - 94(t'u \pm vy')^2 = -10 \end{matrix}} \right\} \dots\dots\dots (6).$$

or, $t_n'^2 - 94y_n'^2 = -10$

The smallest integral values for $v/u = 2143295/221064$, but as fractional

values for t' and y' can be used as shown in (3), to obtain these we solve (5).

Let $v=v'/z$ and $u=u'/z$; then $v'^2-z^2=94u'^2$ (7).

Now let $u^2=pq$ and let 94 =any two factors, then (7) can be made

$$\left. \begin{array}{l} v'+z=p^2 \text{ or } 2p^2 \\ v'-z=94q^2 \text{ or } 47q^2 \end{array} \right\}$$

add and subtract, etc. $v'=p^2+94q^2$ or $2p^2+47q^2$; $z=p^2-97q^2$ or $2p^2-47q^2$; $u'=2pq$(8).

In the right-hand values if $p=5$ and $q=1$, $v'=97$; $z=3$; $u'=10$. There are an infinite number of values but these are the only ones admissible.

(7) $v=97/3$ and $u=10/3$; substituting these along with those of (4) separately in (6) we have $t_n'=2/3$ and $24442/3$; and $t_n'=3946/3$ and $16493426/3$ with those in (4), will make six values for t' , and now in (3) and (2) $x=0, 4, -42, 260, -2714$, and 175462 , etc. The sign=side($2x\pm1$). $y=94, 39, 407, 2521, 26313$.

III. Solution by the PROPOSER.

This problem is suggested by a remark in No. 5, Vol. I.: " $x^2-94y^2=\pm1$; this is the most difficult number under 100."

1. Find initial terms in that infinite series of rational rectangular solids where the edges of each term are in proportion as $2:3:9$, within 1 in the *thickness*.

Let $2x\pm1$, $3x$ and $9x$ be the edges; then $94x^2\pm4x+1=\square=(mx\pm1)^2=m^2x^2\pm2mx+1$. $x=(\pm2m\mp4)/(94-m^2)$.

Say $m=\sqrt{94}=9/1, 10/1, 29/3, 97/10, 126/13, 223/23, 1241/128, 1464/151$, etc.

When $m=$	10	29/3	97/10	223/23	
Then $x=$	4	42	260	2714	
$2x\pm1=$	9	83	521	5427	Thickness.
$3x=$	12	126	780	8142	Width.
$9x=$	36	378	2340	24426	Length.
$\sqrt{94x^2\pm4x+1}=$	39	407	2521	26313	Solid diagonal.

2. Find first term in an infinite series of rational parallelopipeds where the dimensions of every solid are in proportion as $2:3:9$, within 1 in the *width*.

Let $2x$, $3x\pm1$ and $9x$ represent the edges. Then $94x^2\pm6x+1=\square=(mx\pm1)^2=m^2x^2\pm2mx+1$. Whence $x=(2m\mp6)/(94-m^2)$, $m=\sqrt{94}=9, 10, 29/3, 97/10, 126/13$, etc.

$m=29/3$	126/13
$x=$ 24	429
$2x=$ 48	858
$3x\pm1=$ 73	1286
$9x=$ 216	3861
Solid diagonal=	233
	4159

3. Find a term in an infinite series of rational parallelopipeds where the edges are in proportion as 2 : 3 : 9, within unity in *length*.

Let $2x$, $3x$, and $9x \pm 1$ be the edges. $94x^2 \pm 18x + 1 = \square = (mx \pm 1)^2 = m^2x^2 \pm 2mx + 1$. $x = (2m \mp 18)/(94 - m^2)$. Substitute $m = 1464/151$, and $x = 15855$, $2x = 31710$, $3x = 47565$, $9x - 1 = 142694$.

Proof : $31710^2 + 47565^2 + 142694^2 = 153719^2$.

4. Find some term in an infinite series of rational parallelopipeds where the dimensions come within 1 unit in the *thickness* of being in proportion as 3 : 6 : 7.

Let edges be $3x \pm 1$, $6x$ and $7x$. $94x^2 \pm 6x + 1 = \square = (mx \pm 1)^2 = m^2x^2 \pm 2mx + 1$. $x = (2m \mp 6)/(94 - m^2)$.

When $m = 29/3$	$m = 126/33$	
$x = 24$	$x = 429$	
$3x \pm 1 = 144$	$3x \pm 1 = 1286$	etc.
$6x = 144$	$6x = 2574$	
$7x = 168$	$7x = 3003$	
S. d. = 233	S. d. = 4159	

Proof : $73^2 + 144^2 + 168^2 = 233^2$.

5. Find some term in an infinite series of rational rectangular solids where the edges come within 1 unit in the *width* of being in the proportion of 3 : 6 : 7. Let the edges be represented by $3x$, $6x \pm 1$ and $7x$. Then $94x^2 \pm 12x + 1 = \square = (mx \pm 1)^2 = m^2x^2 \pm 2mx + 1$. $x = (2m \mp 12)/(94 - m^2)$. When $m = \sqrt{94} \dots \dots 1464/151$. Then $x = 84258$ or 357870 .

$3x = 252774$	or $3x = 1073610$
$6x - 1 = 505547$	$6x + 1 = 2147221$
$7x = 589806$	$7x = 2505090$
Diagonal = 816911	Diagonal = 3469679

6. Find a term in that infinite series of rational parallelopipeds wherein the edges of every solid are within unity in the *length* of being in proportion to each other as 3 : 6 : 7.

$$(3x)^2 + (6x)^2 + (7x \pm 1)^2 = 94x^2 \pm 14x + 1 = \square = (mx \pm 1)^2.$$

$94x \pm 14 = m^2x \pm 2m$. $x = (2m \mp 14)/(94 - m^2)$. $m = \sqrt{94}$. Now when $m = 29/3$, $x = 60$, $3x = 180$, $6x = 360$, $7x - 1 = 419$.

$$180^2 + 360^2 + 419^2 = 581^2.$$

Also solved by J. H. DRUMMOND.

51. Proposed by H. C. WILKES, Skull Run, West Virginia.

The difference between the roots of two successive triangular square numbers, [i. e. triangular numbers that are also square numbers], equals the sum of two successive integral numbers, the sum of whose squares will be a square number. Demonstrate. Or, if s and t be the roots of any two successive triangular number that are also square numbers, prove that $t - s = 2n + 1$, where $n^2(n + 1)^2 = \square$.

I. Solution by G. B. M. ZERR, A. M., Ph. D., Texarkana, Arkansas.

$$\frac{n(n+1)}{2} \text{ is a square when } n = \frac{(1 + \sqrt{2})^{2m} + (1 - \sqrt{2})^{2m} - 2}{4}.$$